There is no dearth, however, of practical statistical examples and exercises in this interesting book. They are well graded to illustrate the various principles in the text, and should make the work very welcome to instructors of college classes as well as to statisticians in general.

JAMES MCMAHON.

The Mathematical Theory of Population, of its Character and Fluctuations, and of the Factors which Influence them. By G. H. KNIBBS. Appendix A, Volume 1, Census of the Commonwealth of Australia. (Melbourne: Commonwealth Statistician. 1917. Pp. xvi, 466.)

The author says in his foreward that this monograph "aims on the one hand at supplying the elements of a mathematical technique, such as are needed for the analysis of the various aspects of vital phenomena that come under statistical review, and, on the other, at interpreting material made available by the first Census of Australia which has been carried out upon uniform lines and by a central authority." The results of the study "have brought into clearer relief the necessity for recognizing that the variation of any one statistical element affects all other statistical elements, so that the satisfactory reduction of 'crude data' to a common system is by no means an easy undertaking, and the comparability of the statistic of two communities can never be rigorously exact in all particulars."

The scope of this highly technical mathematical analysis may be gleaned from a brief description of its contents. The first eight chapters are devoted mainly to method, such topics as the following being discussed: types of population fluctuations; curve constants and intermediate values; types of curves and their characteristics; group value and integration for statistical aggregates; the place of graphics and smoothing in the analysis of population statistics; conspectus of population characters.

In these chapters primary emphasis is given to the development of mathematical formulas to describe population distributions and the curves which represent them. Among other things discussed are types of population fluctuation, note being taken of the determining factors which secularly influence rate of population increase. A large amount of comparative data on population growths is given for the important countries of the world and the likelihood and consequence of such a rate continuing are pointed

out in detail. Especially worth noting in this part of the volume is the discussion of smoothing statistical data. The fact that general impressions that are often unwittingly or purposely conveyed by a free use of smoothed curves is sufficient justification for quoting briefly from the volume on this point.

There are four principal classes of data to which the process of curve-smoothing is applicable. These may be indicated as follows:

(i.) Frequencies of a phenomenon at successive epochs or during successive periods of time; as, for example, population estimates at given dates and numbers of deaths occurring during successive years.

(ii.) Rates of occurrence of a phenomenon per unit of reference during successive periods; as, for example, birth-rates per thousand of population per annum for successive years.

(iii.) Frequencies in respect of successive values of characters capable of continuous variation; as, for example, the number of persons at each age recorded at a given census.

(iv.) Rates of occurrence of a phenomenon per unit of reference in respect of successive values of characters susceptible of continuous variation; as, for example, rates of mortality per unit per annum during a given decennium in respect of each age.

In all these cases the characteristic of continuous variation is assumed to exist either actually or virtually. Where statistical results are *discontinuous* such a process is, strictly speaking, inapplicable; as, for example, in the tabulation of census population according to birthplace, occupation, or religion. In some cases, however, although the data are strictly speaking discontinuous, the principle may be applied partially; for example, in the case of a tabulation of dwellings according to number of rooms or according to number of inmates. In such cases the character possessed is *progressive without being continuous;* nevertheless, with proper qualifications, the smoothing principle may be applied even to these.

Object of smoothing.—From the foregoing it will be seen that the data to which the smoothing process is strictly applicable are those which may be regarded as functions of a continuous variable. . . . The essence of the matter is that in any instance the data are in the main such as admit of representation by means of a continuous line, or a continuous surface or solid in relation to continuous units of reference. When such representation has been made of the crude results of observation, it is ordinarily found that the line surface or solid exhibits evidence of marked irregularities as between adjacent points or series of points, their general trend, however, suggesting an underlying basis of orderly progression. This progression is, of course, affected by minor influences operating at individual points, and is more or less masked by the paucity of the data on which the representation has been based; thus, suggesting further that were it possible to obtain data of unlimited extent, these irregularities would become neg-

ligible. For this reason the object of the smoothing process may be said to be that of removing these apparently accidental irregularities, and of thus disclosing the basic or ideal uniformity which may be presumed to represent the facts in all their generality.

Justification for smoothing process.—The justifications for the smoothing process may thus be said to be:

(a) That the irregularity does not represent the phenomenon in its generality, since much of the observed irregularity is known *a priori* to be due only to paucity of data;

(b) or that it is known that the phenomenon subject to observation is really regular;

(c) or, again, that the observed data suggest that regularity of trend will not efficiently represent them.

It has been objected that any system of smoothing is, strictly speaking, unwarrantable, since such a process virtually attempts to make the facts accord with more or less questionable preconceptions regarding them. To this view it may be rejoined that if the process were such as to produce results which, though smooth, differed systematically and materially in their distribution from the original observations, the objection would be valid. Where, however, due consideration is given to the relative magnitudes of the original data, and the smoothed results accord therewith as closely as the data will allow when these exhibit a general trend, then the only preconception that can be regarded as operative is the justifiable one that ordinarily natural phenomena do not progress *per saltum*. In this connection it must be noted that where there is distinct evidence at any stage of a cataclysmic disturbance of results, the smoothing process for such points or periods will usually be invalid or not properly applicable. . . .

One of the most cogent justifications for the smoothing process has its warrant in the fact that the recorded results of any statistical observations are necessarily approximative, and hence that the value of the function *recorded* for any given value of the variable is probably not usually more accurate than an estimate based on the recorded values in respect of preceding and succeeding values of the variable. This consideration suggests the idea of weighting successive observations to obtain most probable values, which idea forms the basis of one of the leading methods of adjustment. Again, where the results of the observations are to be employed as guides to future action, it is clear that these results should, as far as practicable, be freed from all fluctuations which may be considered merely accidental, and thus unlikely to be reproduced in future experience. This is of considerable importance in connection with the construction of mortality and sickness, superannuation, and similar tables to be used in the computation of rates of premium, and for the conduct of valuations.

Later chapters are given over to description of the Australian population according to sex and age; masculinity; natality; nuptiality; fertility and fecundity and reproductive efficiency; mortality; migration, etc. This part of the discussion is likewise mathe-

matical and unique in studies of population. Nowhere else, so far as the writer's knowledge goes, can there be found, for instance, such a fundamental and critical analysis of error, as is encountered in statistics of birth, marriage, and divorce, given in this monograph. A unique feature of the discussion is the treatment of migration. Under this head are discussed such topics as correlation, owing to migration, between age and length of residence, periodic fluctuations in migration, migration and age, etc.

It is impossible within the short compass of a review adequately to deal with the scope and merits of this monumental piece of statistical technique. The author's concluding words, however, may be quoted to indicate his own conception of its purpose and of "the larger aim of population statistic."

> At present there exists a large and accumulating mass of unanalysed material. Numerical data have in many instances already become a burden, and in other cases threaten to become one. But when their significance has been penetrated they seem no longer tedious; they have been transformed into illuminating and interesting facts.
>
> Here, however, we need a word of warning. The problem of all so-called knowledge is to subsume what we know—or think we know—under suitable elementary conceptions, conceptions, in fact, that are within our intellectual grasp, and that we can mentally handle. As in physics the Boyle-Charles gaseous laws, the molecular law of equal numbers in equal volumes at equal pressures and temperatures, and the conception of mass as independent of velocity, are but crude statements of the actual facts, so crude that their elementary simplicity entirely disappears when necessary qualifications are made, so likewise does a deeper knowledge of statistic reveal that relations subsisting among crude data are subject to corrections that, not infrequently, are very elaborate. The more simple and obvious of these relations constitute a kind of rough frame-work about which more subtle and accurate conceptions may cluster, or, to change the figure, they are a skeletal foundation on which the body of justly conceived statistic is to be built up.
>
> Anyone who has seriously reflected upon the facts of the last ten decades must realise that, within the next ten, tremendous problems will arise for solution and these will touch fundamentally the following matters, viz.:
>
> (i.) The multiplying power of the human race;
> (ii.) The organic constitution of Nature and the means at human disposal for avoiding the incidence of its unfavorable aspects; *i.e.*, eugenics in its wider sense;
> (iii.) The enhancing of the productivity of Nature, and the limits of its exploitation;
> (iv.) The mechanism of the social organism, and the scheme of its control;
> (v.) Internationalism and the solidarity of humanity.

For the adequate study of these matters, not only will the mere technique of the collection and analysis of statistic require to be much advanced, but the popular opinion as to the value of the effort will also have to progress. Given, however, an intelligent public opinion, as to the utility of statistical inquiries, there would be some ground for hope that the great questions, the analysis of which would throw light upon human destiny, could be properly attacked. It is for educational departments, worthy of the name, to create such opinion by the mechanism of their systems, in order that each human being should be sufficiently interested to cordially co-operate, by accurately furnishing the necessary data in the taking of a census of population or wealth. Census-taking is a costly operation, but it is the foundation of all branches of statistic that have a direct human interest. Its value and the facility of using it would be immensely increased if it were meticulously accurate. The importance of technique and of precision, matters apparently of little moment, can be rightly estimated only when the ultimate aim of all statistical inquiry is realised to be 'the study of man's destiny' as the denizen of a world of limitations.

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NEW BOOKS

HOFFMAN, F. L. *Army anthropometry and medical rejection statistics.* (Newark, N. J.: Prudential Ins. Co. 1919. Pp. 114.)

MACDONALD, A. *Anthropometry of soldiers.* Reprinted from the *Medical Record,* Dec. 14, 1918. (New York: William Wood & Co. 1918. Pp. 17.)

TRIVETT, J. B. *The official year book of New South Wales, 1917.* (Sydney: Bureau of Statistics. 1918. Pp. 843. 2s. 6d.)

Census of war commodities. Statistics of leather. (Washington: Bureau of the Census. 1918. Pp. 13.)

Eidgenössisches statistisches Bureau. Statistisches Jahrbuch der Schweiz, 1917. (Bern. 1918. Pp. 371.)

Statistical year book of Quebec, 1918. Fifth year. (Quebec: Bureau of Statistics. 1918. Pp. xiii, 565.)